# Zeeman Effect

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**Abstract**

This experiment investigates Compton scattering using a scintillation counter, photomultiplier, and multi-channel analyzer to validate key predictions of quantum electrodynamics. By calibrating our system with a $^{137}$Cs source, we measured the scattered photon energy at various angles and analyzed the results to verify the Dirac theory of the electron and the Klein-Nishina formula. Our calculated electron mass, $m = (9.51 \pm 0.15) \times 10^{-31}$ kg, is within 4.45% of the accepted value, confirming the expected energy-momentum relationship. Additionally, we determined a proportionality constant of $c = (2.33 \pm 0.27)$ between measured counts and the Klein-Nishina cross-section, and our plot of measured counts and weighted Klein-Nishina cross-section versus Compton scattering angle demonstrates strong agreement with theoretical predictions. These findings support the wave-particle duality of photons and the quantum mechanical nature of electron-photon interactions.

## Introduction

In November 1922, Arthur Holly Compton sketched a diagram for his students at Washington University in St Louis, Missouri. From the left, a photon, or "incident quantum," collides with a stationary electron, which causes the pair to recoil and conserve momentum and energy. That was the first time Compton shared his breakthrough formulation of X-ray scattering from electrons.[1] In 1927, he was given the Nobel prize for this discovery, now his namesake. The Compton effect was groundbreaking in establishing the wave-particle nature of the photon, providing a relationship between the momentum of a particle and its wavelength.[2] In this lab, we will study Compton scattering through gamma rays, using a scintillation counter, photomultiplier and multi-channel analyzer. Recording Compton scattering peaks at different scattering angles, we will be able to verify the Dirac theory of the electron and the Klein-Nishina formula.

# 1 Experiment setup and procedures

Following the steps outlined in the lab manual [6], we first proceed to calibrate our system using a separate $^{137}$Cs source mounted 6 inches from the detector. Closing the shutter of our main source, we take measurements of the final charge $Q$ in our photomultiplier as a function of tube voltage $V$. Knowing that $Q \propto V^{N/2}$, with $N$ the number of dynodes, we are able to plot $log\,Q$ versus $log\,V$, and perform a linear fit, giving roughly a straight line with slope $N/2$ (fig. 1).

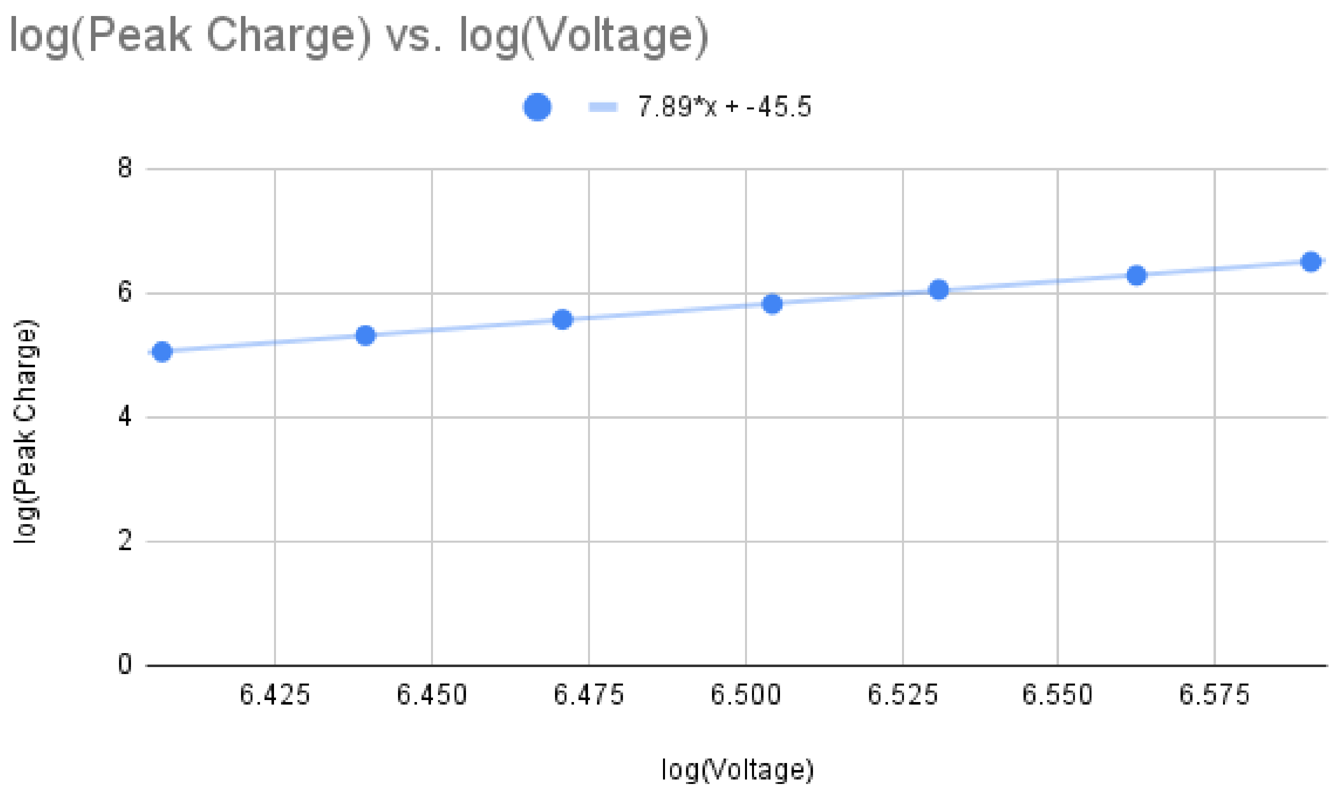


Figure 1: Calibrating our system by verifying the number of dynodes in our photomultiplier

Our figure shows a slope of $7.89 = N/2$, therefore $N$ is about 15.78, or 16 dynodes, which is about the number of dynodes specified by the manufacturer. [4]

We continue our calibration with our separate $^{137}$Cs source, now looking to position the peaks appropriately in our multi-channel analyzer. Since we expect the energy of the peaks to drop with increasing angle, we set our photomultiplier voltage such as to give us the most range in energy. We look at the analyzer's channels for different photomultiplier voltages, and find that setting to 738 V gives us the most range in energy, placing our Compton scattering peak furthest to the right without hiding it (fig. 2).

Having set our photomultiplier voltage, we proceed to calibrate our multi-channel analyzer by giving it the energy associated with each peak (fig. 2). We know our upper peak to be 661.7 keV, and our lower peak to be 32.19 keV. [3].

Finally, we obtain a spectrum of Compton scattering peaks from our multi-channel analyzer at 11 different angles, using our main $^{137}$Cs source scattering off the aluminum rod. Since there may be a slight thermal drift of the high voltage and the gain of the amplifier, we used the sealed source for intermediate calibrations before every measurement. The

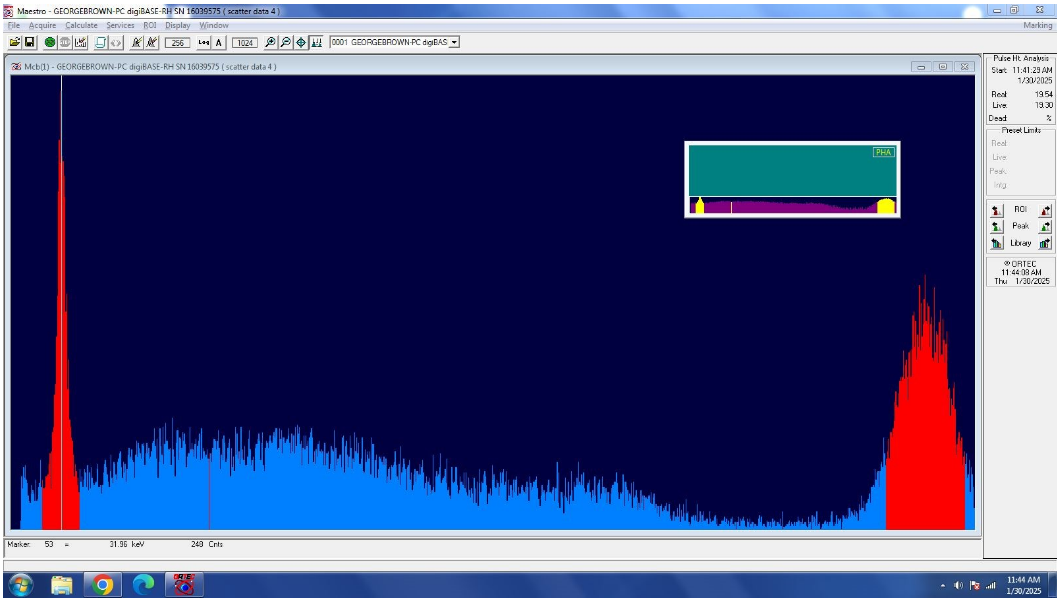

Figure 2: Compton scattering peaks of a $^{137}$Cs source with the photomultiplier set to 738 V

multichannel analyzer automatically fits a Gaussian function and subtracts background energy.

## 2 Results Measured

Having calibrated our setup, we proceed to verify the Dirac theory of the electron and the Klein-Nishina formula. The Dirac theory of the electron required that scattered photons lose energy upon scattering by electrons, according to the following formula:

$$\frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\theta)}{mc^2}$$

where $E$ is the incident photon energy and $E'$ is the scattered photon energy.

Klein and Nishina calculated the differential cross section, and showed that:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{(E'(\theta)}{E}\right)^2 r_0^2 \left[\frac{E}{E'(\theta)} + \frac{E'(\theta)}{E} - \sin^2(\theta)\right]$$

### 2.1 Finding electron mass

Since we know that the incident and scattered photon energy are linearly related, we take advantage of this fact to find $m$, the mass of the electron. Measuring $E'$ for 11 different angles, we plot our values in fig. 3. Our error bars are calculated from the multi-channel analyzer manual ([5] pg. 71).

### 2.2 The Klein-Nishina formula, to a constant of proportionality

Our multi channel analyzer automatically subtracts background noise from our gaussian function, and returns the net area under each our peaks with a specified error. We normalize the net peak area by taking the ratio of the longest time

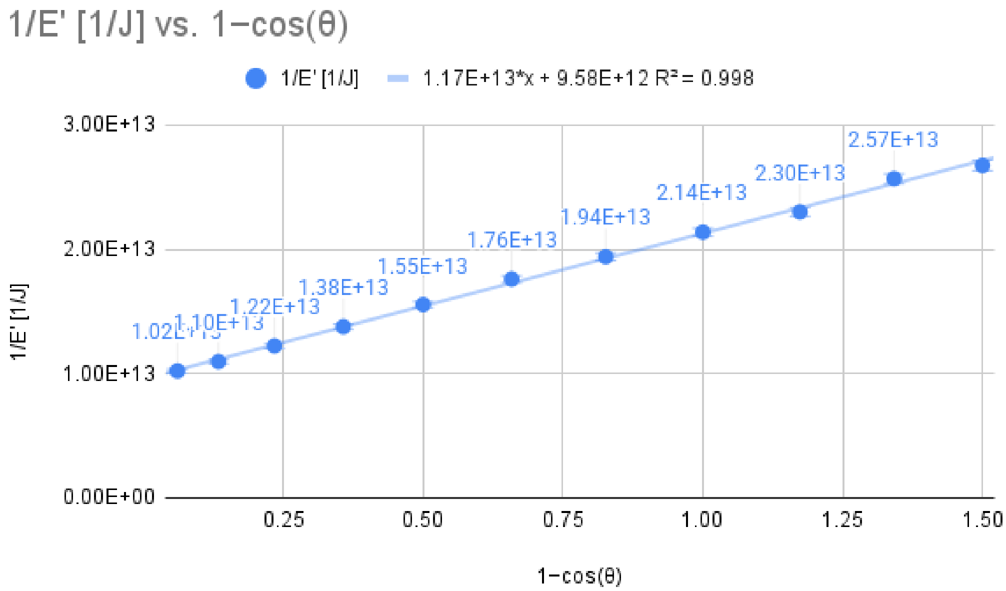


Figure 3: Finding the mass of the electron through Compton scattering peak energies

to obtain the peak over the time to obtain the peak.

| angle $\theta$ [degrees] | normalized net peak area [counts] | peak area error [+-counts] | theoretical $\frac{d\sigma}{d\Omega}$ |
|---|---|---|---|
| 20 | 7598.735984 | 877.2426096 | 6.40597E-30 |
| 30 | 11489.30488 | 650.2883378 | 5.22124E-30 |
| 40 | 7485.767854 | 701.3692404 | 3.91423E-30 |
| 50 | 7539.105483 | 768.7528054 | 2.90415E-30 |
| 60 | 5742.288788 | 590.3072874 | 2.20135E-30 |
| 70 | 5100.229438 | 598.3278871 | 1.73296E-30 |
| 80 | 4238.038254 | 683.0537845 | 1.47788E-30 |
| 90 | 3976.634037 | 595.4125372 | 1.32033E-30 |
| 100 | 4400.55815 | 676.4724891 | 1.25542E-30 |
| 110 | 4114.988522 | 696.7541336 | 1.18271E-30 |
| 120 | 2175 | 582 | 1.20503E-30 |

We ignore the counts for the scattering at 20 degrees while finding our normalization constant, since it had too much forward scattering.

# 3 Data Discussion and Analysis

## 3.1 Electron Mass Calculations

Plotting $\frac{1}{E'}$ versus $1 - \cos(\theta)$ and performing a linear regression gives us a slope of $\frac{1}{mc^2}$ and an intercept of $\frac{1}{E}$. In fig. 3, we see that our slope is $a = (1.17 \pm 0.18) \times 10^{13}$, which gives $m = \frac{1}{ac^2} = (9.51 \pm 0.15) \times 10^{-31}$ kg. This value has an error of 4.45% relative to the theoretical mass of the electron $9.11 \times 10^{31}$ kg. Looking at our intercept, our value of $(9.58 \pm 0.15) \times 10^{12}$ J$^{-1}$ has an error of 1.55% relative to the theoretical $\frac{1}{E} = 9.43 \times 10^{12}$ J$^{-1}$.

### 3.2 Verifying the Klein-Nishina formula

We know that $\frac{d\sigma}{d\Omega}$ is proportional to our counts, so we calculate a constant of proportionality and show that the relative scattering cross section as a function of angle is proportional to the Klein-Nishina formula. To find the constant of proportionality $c$ between measured counts and the Klein-Nishina formula, we plot the values of $\chi^2$ versus different $c$ values (fig. 4).

$$\chi^2 = \sum_{i=1}^{n} \frac{(i^{th} \text{ Peak Area} - c \cdot \frac{d\sigma}{d\Omega}_i)^2}{\frac{d\sigma}{d\Omega}_i}$$

Seeing that the minimum is $(2.33 \pm 0.27)$, we plot the new weighted Klein-Nishina formula and our counts versus the Compton scattering angle in fig. 5, and find a strong correlation.

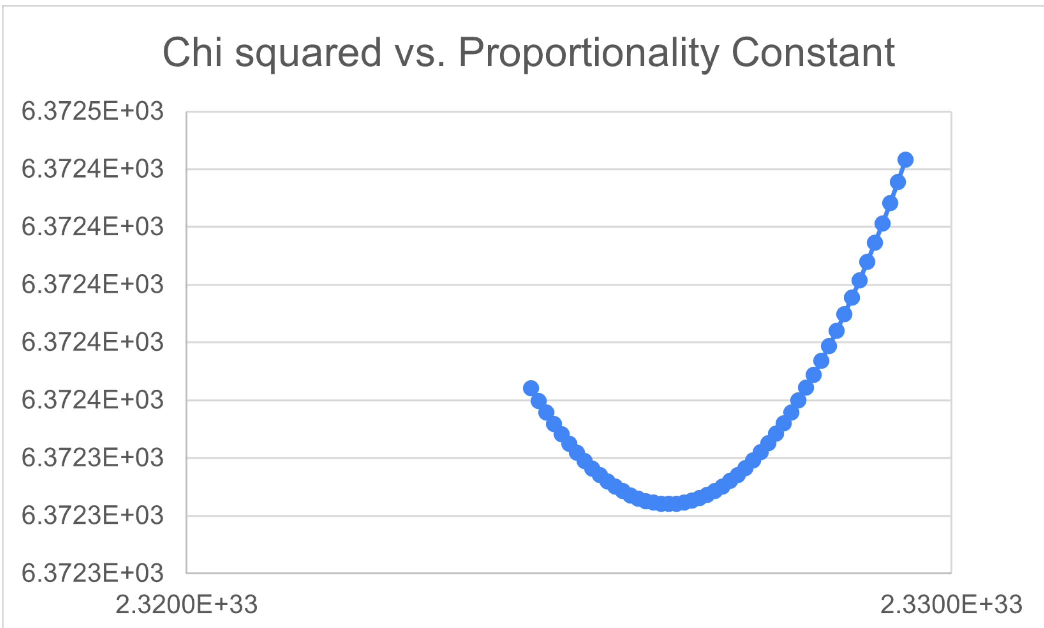


Figure 4: Finding the minimum $\chi^2$ to determine the constant of proportionality between measured counts and the Klein-Nishina formula

# Conclusion

In this experiment, we successfully analyzed Compton scattering using a scintillation counter, photomultiplier, and multi-channel analyzer. By calibrating our system with a $^{137}$Cs source, we determined the appropriate photomultiplier voltage and verified the number of dynodes in our photomultiplier tube. Using these calibrated settings, we recorded Compton scattering peaks at eleven different angles and used this data to test two key theoretical predictions: the Dirac theory of the electron and the Klein-Nishina formula.

From our measured values of scattered photon energy at different angles, we plotted $\frac{1}{E'}$ versus $1-\cos(\theta)$ and determined the electron mass to be $m = (9.51 \pm 0.15) \times 10^{-31}$ kg, which is within 4.45% of the accepted value of $9.11 \times 10^{-31}$ kg. Additionally, our intercept value of $\frac{1}{E} = (9.58 \pm 0.15) \times 10^{12}\ \mathrm{J}^{-1}$ closely matched the theoretical prediction with an error

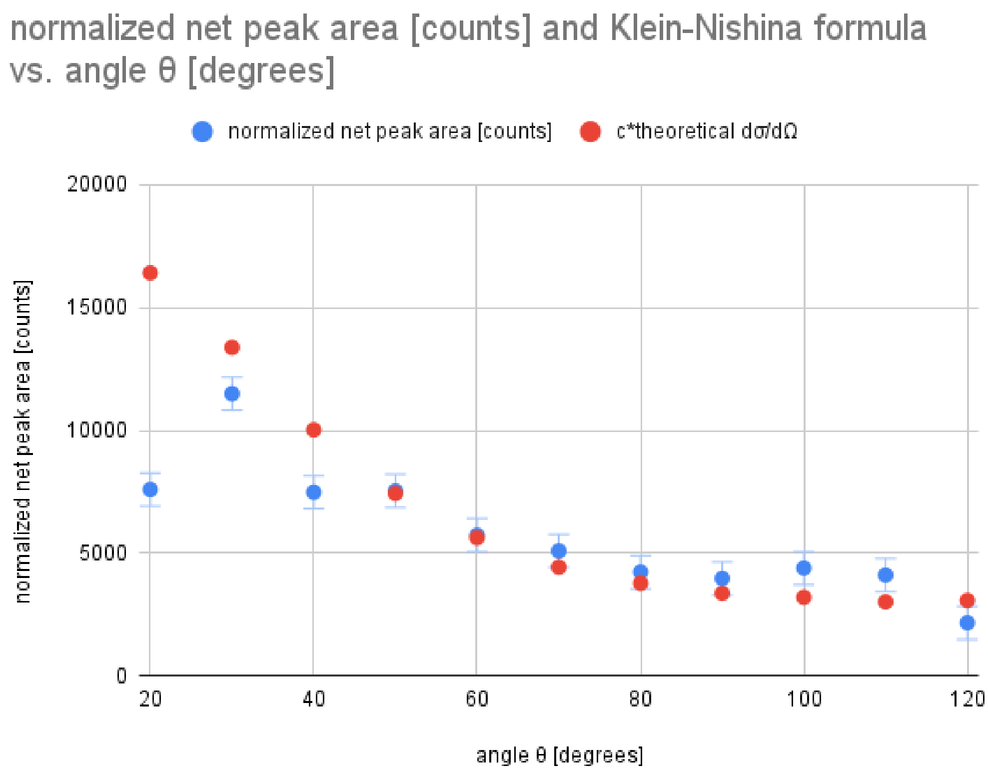


Figure 5: Comparing counts obtained and the Klein-Nishina theoretical differential cross-section shows a correlation

of only 1.55%. These results confirm that Compton scattering follows the expected energy-momentum relation derived from the Dirac theory.

Furthermore, by normalizing the net peak area of our measured Compton scattering peaks and comparing them with the Klein-Nishina differential cross-section, we found a proportionality constant of $c = (2.33 \pm 0.27)$. Our data exhibited a strong correlation with the Klein-Nishina theoretical model, validating its prediction of the angular dependence of Compton scattering.

Overall, this experiment effectively demonstrated the fundamental principles of quantum electrodynamics by verifying the mass of the electron through Compton scattering and confirming the Klein-Nishina formula. While minor sources of error, such as thermal drift and detector resolution, may have influenced our results, our findings align closely with established theoretical values, reinforcing the wave-particle duality of photons and the quantum mechanical nature of electron-photon interactions.

## References


[1] R. H. Stuewer, Ann. Phys. (Berlin) 512, 975 (2000). https://doi.org/10.1002/andp.200051211-1216

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[3] X-Ray Data Booklet, https://xrd.lbl.gov

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[5] ORTEC MAESTRO A65-BW Software User's Manualhttps://www.ortec-online.com/-/media/ametekortec/manuals/a/a65-mnl.pdf?la=en&revision=f2f3ed3f-fa2d-4185-9301-7d480b0a6955

[6] U.C. Santa Cruz Advanced Physics Laboratory Manual